From March 2011 *High Frequency Electronics*

Clarifications on Mixer Math

We received some valuable comments regarding the January issue's tutorial. "The Mathematics of Mixers: Basic Principles."

Linear or Nonlinear?

The first comment referred to the statement that mixers are nonlinear circuits, and is from Alan Victor.

"Saw you mixer tutorial and one point that grabbed my attention is the 'Mixers are nonlinear circuits'. Whoa! Mixing is quite real in linear systems and the linear mixing function is readily seen by multiplication of linear transconductance with a time varying signal voltage. Your cosine product equation in the second column of the page 34 shows this concept. See also Steve Maas' text on the same, "An Improved Linear Mixer: The FET Resistive Mixer."

Dr. Victor is, of course, correct. The statement in question was taken directly from Carson (Ref. [3] in the article), but should have been qualified—it only applies to mixers such as the common diode double balanced type, which use switching action to achieve an approximation of multiplication. A discontinuity such as on-off switching is certainly non-linear, but it is not the only means of achieving mixing action.

The equation referred to is the trig identity:

$$\cos(\omega_1)\cos(\omega_2) = [\cos(\omega_1 + \omega_2)]/2 + [\cos(\omega_1 - \omega_2)]/2$$

Readers will observe that a mixer based solely on this equation will not perform a simple frequency translation, since there are both sum and difference terms on the right side. But this is easily handled by an expanded analysis that describes an "image-reject mixer" with an output at a single frequency. This is described by a related trig identity:

$$\sin(\omega_1 + \omega_2) = \sin(\omega_1)\cos(\omega_2) + \cos(\omega_1)\sin(\omega_2) \quad (1)$$

First, remember that the sine function is identical to a cosine function shifted by π/2 radians, or 90º.

$$\cos(\pi/2 - \omega) = \sin(\omega)$$

Thus, the left side of eq. (1) is our desired output, the sum of the two input frequencies. Although shown as a sine function, it is a single signal and the relative phase is unimportant.

Next, note that the right side of eq. (1) describes two mixers, one with a π/2 shift in ω_1 and the other with a π/2 shift in ω_2 (sin vs. cos functions). The combined outputs of these two mixers produce only the sum of the input frequencies, rejecting the unwanted difference frequency.

The above process achieves the translation of f_1 by the "distance" of f_2. The mathematics is linear, and a circuit that accomplishes the two instances of multiplication will also be linear.

Order of Harmonics

The next comment came from Al Martin of Segue Services, Inc.:

"I enjoyed the tutorial in the Jan. 2011 on mixers. I'm wondering if there is an error, or if I'm misunderstanding something. At the bottom of the third paragraph is the statement, "...order of the harmonics, from zero (fundamental) to infinity. Instead of 'zero', should that be 'one'?

"Thank you for your help. I greatly enjoy your tutorial articles, and look forward to them each month!"

Mr. Martin spotted another error in rephrasing notes from one of the references. Yes, the fundamental is the first order harmonic. The zero-order term would be any DC component on the signal.

Also note that this comment refers to nonlinear behavior, whether due to an intentional discontinuity such as a switching-type mixer, or deviations from ideal behavior in linear mixers. The harmonics correspond to the terms in the Fourier series that results from the transfer function of the circuit. From our Signals & Systems courses, we know that the zero order term is a constant that represents DC in real electronics, while the remaining terms represent the amplitudes of the harmonics.

Additional Mixer Types

There are several other common types that readers may wish to examine in their own further reading—single-ended (unbalanced) mixers, FET transconductance mixers, BJT mixers (common in ICs) and frequency doubling mixers are used in various applications. Devices with special performance features may also be of interest, such as varactor diodes, Gunn diodes, or tunnel diodes. Also, the principles that make mixers operate are involved in the analysis of spurious responses and intermodulation distortion.

Final Comment

Our brief tutorials are not intended to be in-depth examinations of a particular topic, and their brevity sometimes makes it difficult to provide enough detail in the explanations. When the information is not clear, feedback such as the questions and comments noted above is greatly appreciated.